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Towards Resilient Tracking in Autonomous Vehicles

A Distributionally Robust Input & State Estimation Approach

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Introduction - Problem & Solution

Image Source: Freepik

The Problem

- ➔ Safety
- ➔ State Data
- ➔ Noisy Measurements

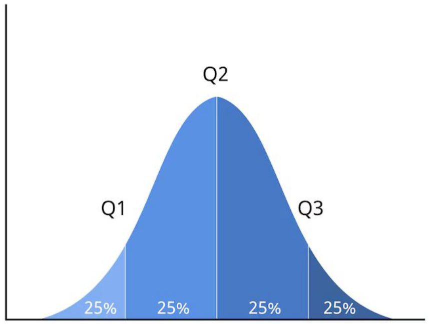
The Solution

- ➔ Fuse Data
- ➔ Estimate State Using Model
- ➔ Ensure Reliability

Baseline - Input & State Estimation (ISE)

Core Idea

- ➔ Joint Estimation
- ➔ State & Unknown Input
- ➔ Enhance Prediction



Key Limitation

Sensitive to:

- ➔ Non-Linearity
- ➔ Non-Gaussian Noise
- ➔ Outliers

Image Source: Freepik

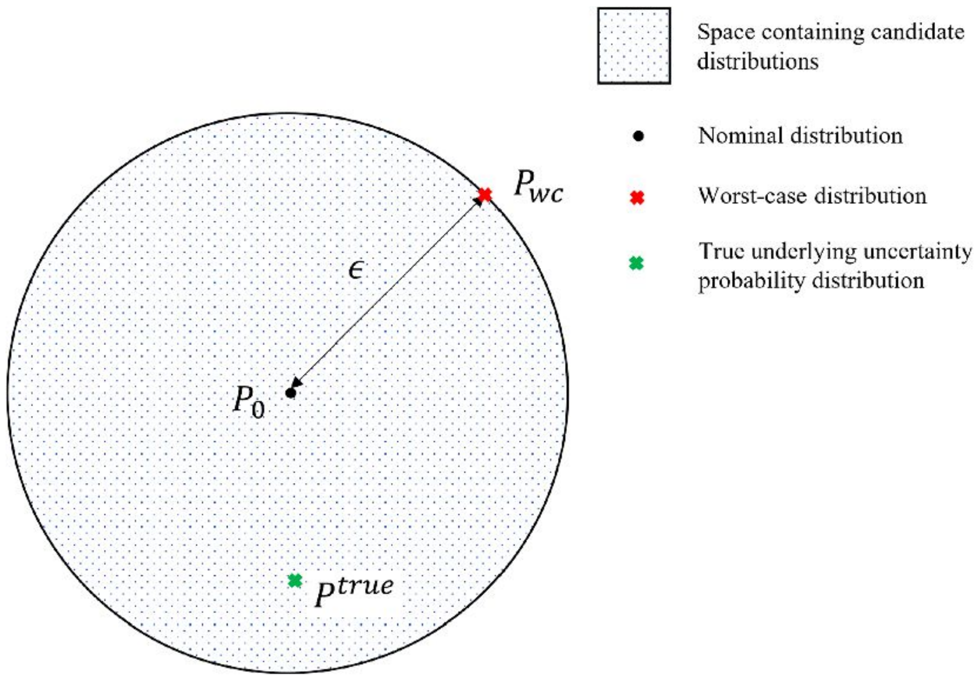
Baseline - Distributionally Robust Estimation (DRE)

Core Idea

- ➔ Robustness
- ➔ Deviating Noise Distributions
- ➔ Ambiguity Sets
- ➔ Worst-Case Optimization

Key Limitation

- ➔ Sensitive to Outliers
- ➔ Ignores Unknown Inputs



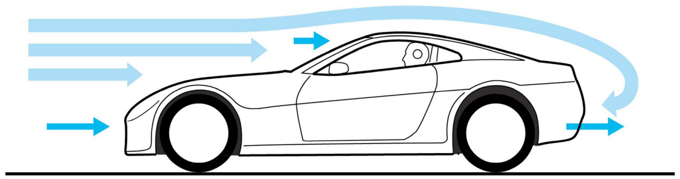


Image Source: Researchgate.net, Gran-Turismo

Problem Formulation

System Model

unknown input

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}_k \boldsymbol{x}_k + \boldsymbol{B}_k \boldsymbol{u}_k + \boldsymbol{G}_k \boldsymbol{d}_k + \boldsymbol{w}_k$$

state vector

known input

uncertain process noise distribution

$$\boldsymbol{y}_k = \boldsymbol{C}_k \boldsymbol{x}_k + \boldsymbol{v}_k$$

sensor measurements containing outliers

uncertain measurement noise distribution

Building Block 1: Unknown Input Estimation

Unknown Inputs

actual measurement

$$\hat{\boldsymbol{d}}_{k-1} = \boldsymbol{M}_k(\boldsymbol{y}_k - \boldsymbol{C}_k\hat{\boldsymbol{x}}_k^-)$$

unknown input

gain matrix

predicted value

Image Source: Freepik

Building Block 2: Distributionally Robust Estimation

Uncertain Noises

worst-case scenario

$$\min_{\phi(\cdot)\in\mathbb{H}'_y} \max_{\mathbb{P}(\boldsymbol{x}_k,\boldsymbol{y}_k|\boldsymbol{y}_{k-1})\in\mathbb{F}'} \mathbb{E}_{\mathbb{P}(.,.|.)}\left[\boldsymbol{x}_k - \phi(\boldsymbol{y}_k)\right]\left[\cdots\right]^\top$$

best performance estimator

ambiguity set

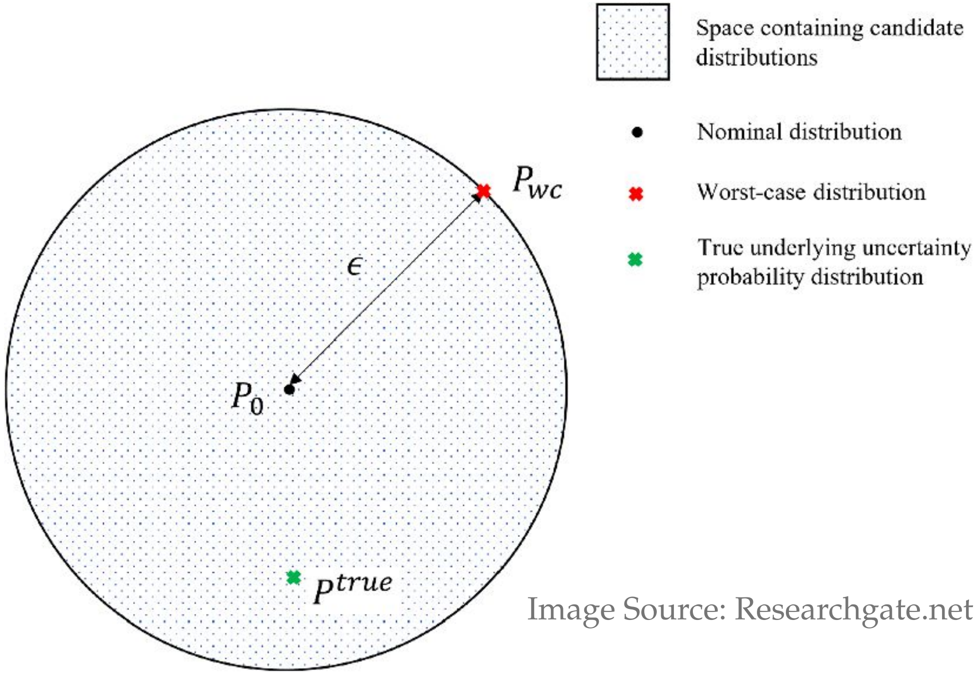


Image Source: Researchgate.net

Building Block 3: Robust Update

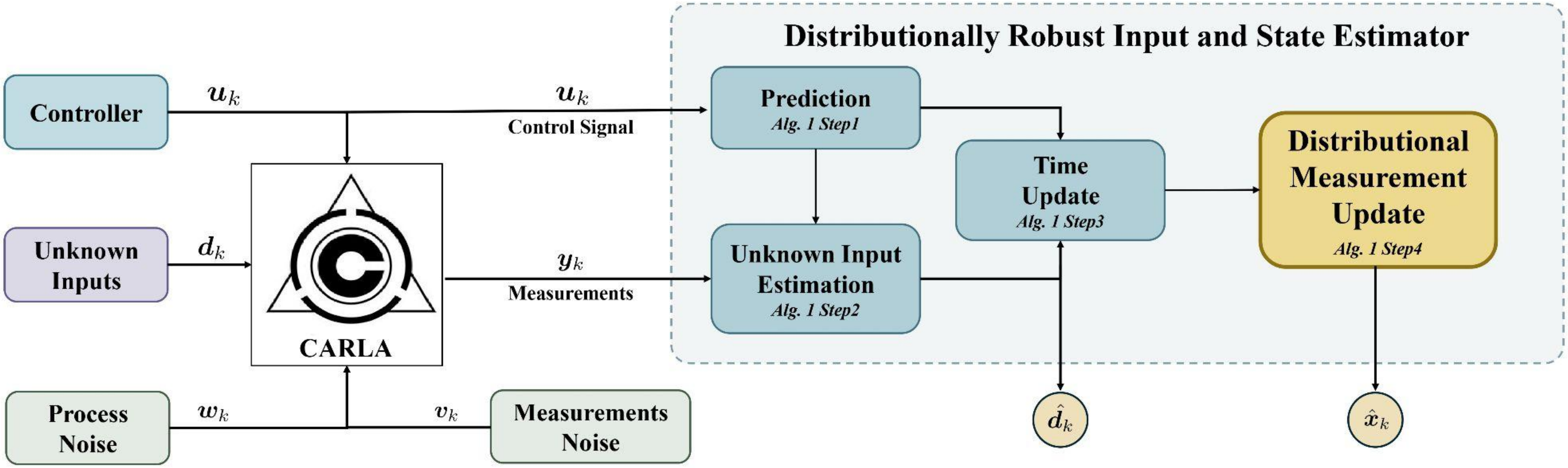
Outliers

measurement residual

$$\psi(\mu) = \begin{cases} -K, & \mu \leq -K \\ \mu, & |\mu| < K \\ K, & \mu \geq K \end{cases}$$

tuning threshold

Block Diagram - DRISE Framework



The DRISE Algorithm Cycle

Key Steps

1. **Prediction**

$$\hat{\boldsymbol{x}}_k^- = \boldsymbol{A}_{k-1}\hat{\boldsymbol{x}}_{k-1} + \boldsymbol{B}_{k-1}\boldsymbol{u}_{k-1}$$

2. **Input Estimation**

$$\hat{\boldsymbol{d}}_{k-1} = \boldsymbol{M}_k(\boldsymbol{y}_k - \boldsymbol{C}_k\hat{\boldsymbol{x}}_k^-)$$

3. **Time Update**

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_k^- + \boldsymbol{G}_{k-1}\hat{\boldsymbol{d}}_{k-1}$$

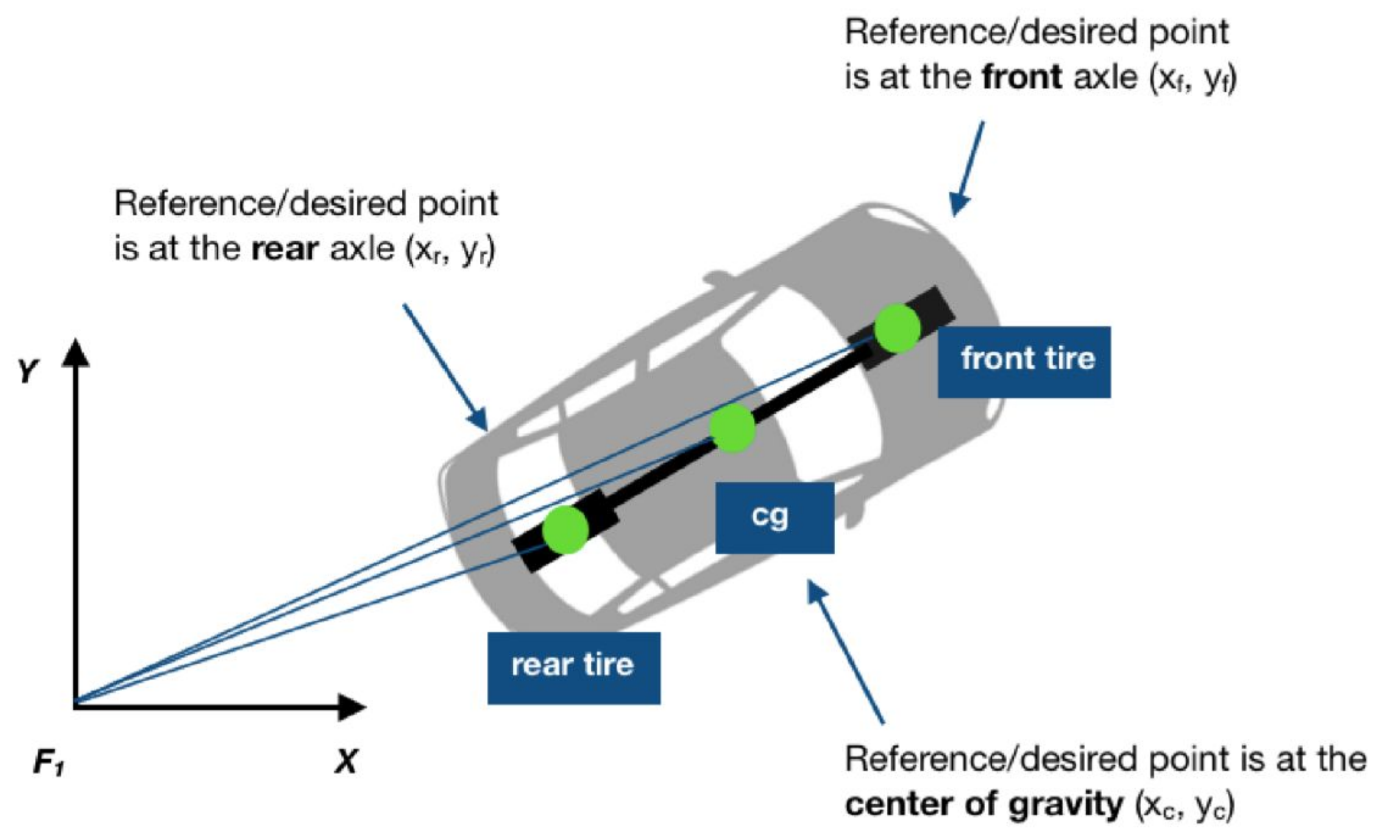
4. **Robust Measurement Update**

$$\hat{\boldsymbol{x}}_k \leftarrow \hat{\boldsymbol{x}}_k + \boldsymbol{L}_k\psi_k(\boldsymbol{S}_k^{-1/2}\boldsymbol{s}_k)$$

Notation

- $\boldsymbol{M}_k$: Input est. gain.
- $\boldsymbol{L}_k$: Robust gain involving ambiguity sets.
- $\psi_k(\cdot)$: Influence function.
- $\boldsymbol{s}_k = \boldsymbol{y}_k - \boldsymbol{C}_k\hat{\boldsymbol{x}}_k$: Innovation.
- $\boldsymbol{S}_k$: Robust innovation covariance.

Simulation Setup



Simulation Settings

- **Model:** Kinematic Bicycle (LTV)
- **States $\boldsymbol{x}_k$:** Pos, Yaw, Vels
- **Input $\boldsymbol{u}_k$:** Steering, Accel
- **Unknown Input ($\boldsymbol{d}_k$):** Time-Varying Signal
- **Noise:** Proc ($\boldsymbol{Q}_k$), Meas ($\boldsymbol{R}_k$)
- **Outliers/Deviations:** Included in Tests
- **Comparison:** KF, ISE, DRE

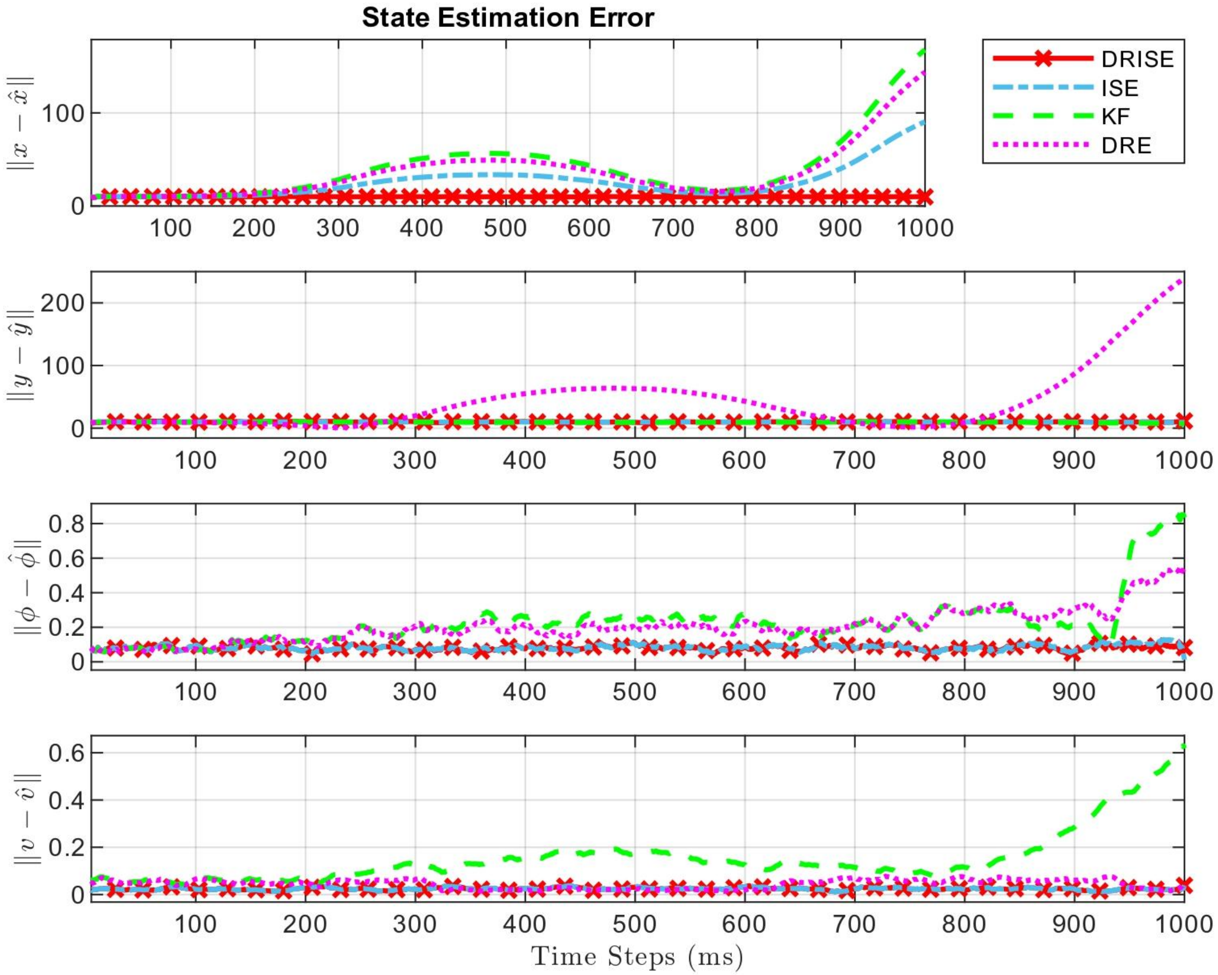
$$\boldsymbol{d}_k = [\text{sign}(\sin(0.005k))]\,[1, 10]^\top$$

CARLA Simulation Environment

Testing in CARLA Simulator

- Open-source, high-fidelity simulator for AV research.
- Provides realistic urban environments, sensors, and physics.
- Challenging testbed for evaluating estimator performance under uncertainty.

Results: State Estimation Error

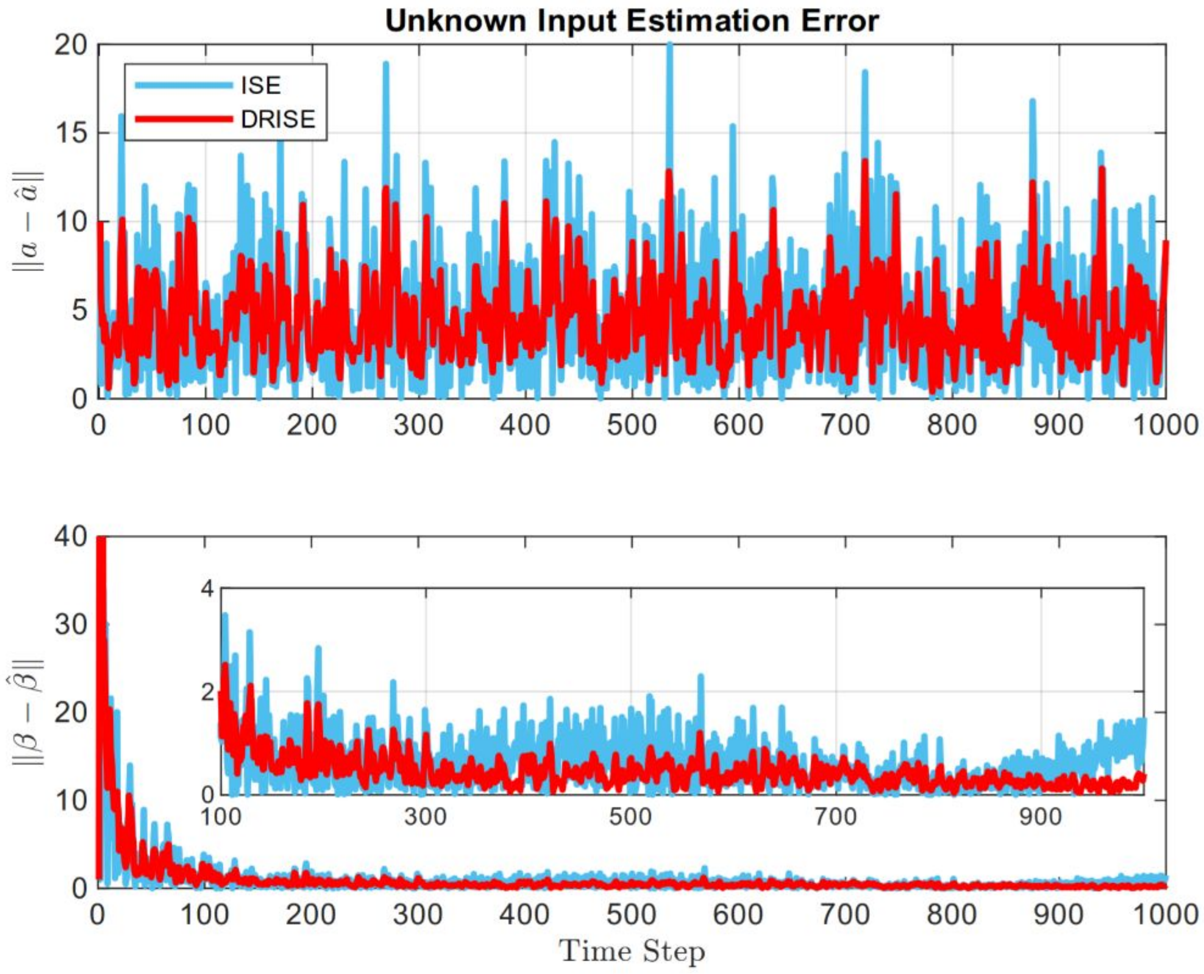


Analysis

- **DRISE:** Lowest Error
- **KF:** Highest Error/Divergence
- **ISE/DRE:** Moderate Error

Method	$RMSE(\hat{\boldsymbol{x}})$
DRISE	14.21
ISE	29.30
DRE	47.37
KF	69.23

Results: Unknown Input Error

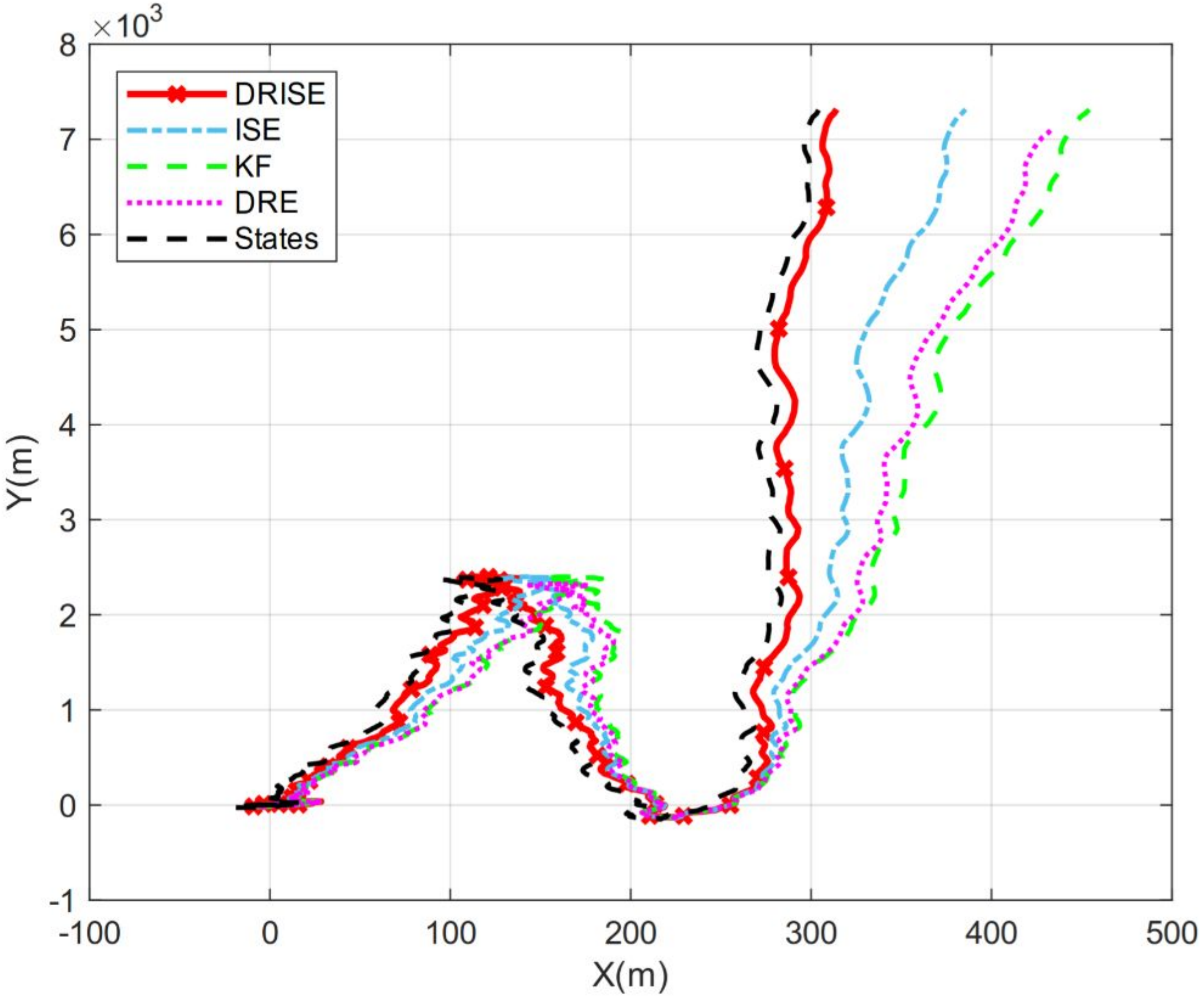


Analysis

- **DRISE:** Lower Error
- **ISE:** Higher Error
- **Benefit:** Robustness Aids Input Est.

Method	$RMSE(\hat{\mathbf{d}})$
DRISE	7.48
ISE	8.40
DRE	-
KF	-

Results: Trajectory Tracking



Analysis

- **DRISE:** Best Tracking
- **Others:** Show Drift
- **Link:** Accurate Est. $\rightarrow$ Better Tracking

Conclusion

DRISE Performance

- **Superior Accuracy**: Lowest State/Input Errors (RMSE)
- **Robustness Confirmed**: Best performance under combined noise/outlier/input challenges
- **Practical Benefit**: Enables Most Accurate Trajectory Tracking

Benchmark Limits

- **KF:** Sensitive to ALL challenges
- **ISE:** Sensitive to Noise/Outliers
- **DRE:** Sensitive to Unknown Inputs/Outliers

Thank you!

Questions?